MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2017/2018

PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

6 JUNE 2018
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 2 pages excluding the cover page and appendix.

2. Answer **all FIVE** questions. Each question carries equal marks and the distribution of the marks is given.

3. Write all your answers in the Answer Booklet provided.

Answer All Questions.

Question 1 [10 marks]

a. Find the following limits.

i. $\lim_{x\to\infty} \frac{x^6 - x^4 + x^2 - 1}{7x^6 + 4x^3 + 10}$ (2 marks)

ii. $\lim_{x\to 0} \frac{\tan x - x}{\sin x}$ (2 marks)

iii. $\lim_{x\to -3} \frac{\sqrt{2x+22} - 4}{x+3}$ (3 marks)

b. Find b such that the function below is continuous.

$$f(x) = \begin{cases} 2x^2 + b, & x \geq -1 \\ -x^3, & x < -1 \end{cases}$$

(3 marks)

Question 2 [10 marks]

a. Differentiate the following function.

i. $y = e^{3x} \ln(4x - 1)$ (2 marks)

ii. $x^2 y^3 - y^2 - 6x = 0$ (3 marks)

b. Integrate $\int x(\ln x)^2 dx$ based on the integration by part method. (5 marks)

Question 3 [10 marks]

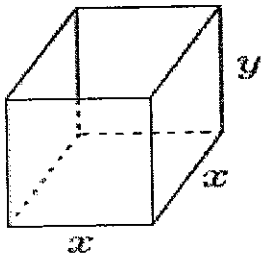
a. Given $f(x) = x^4 - 8x^3$.

i. Find the critical numbers and the intervals on which $f(x)$ is increasing and/or decreasing. Hence, find the maximum and/or minimum point(s). (4 marks)

ii. Find the x-coordinate(s) of inflection point(s) if any. (2 marks)

Continued...

b. An open rectangular box with square base is to be made from 48 ft^2 of material. Find the value of x and y that enable the box to hold the largest possible volume.



(4 marks)

Question 4 [10 marks]

a. Sketch the curve $y = 6 - x - 2x^2$ and the line $y = 4x + 3$.

(1 mark)

i. Find the coordinates of the points of intersection of the line and the curve.

(1 mark)

ii. Find the area of the region enclosed by the line and the curve.

(3 marks)

b. The region bounded by $x = y^2 - 6y + 10$ and $x = 5$ is rotated completely about the y-axis. Find the volume of the solid formed.

(5 marks)

Question 5 [10 marks]

a. Find the general solution for the separable differential equation $y' = \dfrac{x^2 y - y}{y + 1}$.

(4 marks)

b. Find the unique solution for the second order differential equation

$$y'' - 5y' + 6y = 0 \text{ where } y(0) = 1 \text{ and } y'(0) = 1$$

(6 marks)

End of Page

APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

B. Basic Integration Formulas

$\int cf(x)dx = c\int f(x)dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$